

THE APPLICATION OF THE BRF SYSTEM TO
SOME SUPERCONDUCTING MAGNET DESIGN PROBLEMS*R. B. Meuser
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

ABSTRACT

The Berkeley Remote Facility (BRF) system -- affected through a system of teletype terminals linked to the LBL computers -- has been used to solve a large number of magnetic-field problems associated with the design and analysis of superconducting beam-transport magnets. The limitations of the BRF system are severe: total storage, 1000; 10 subscripted variables; no integer or complex arithmetic; no function or subroutine subprograms except those in its Spartan library; and a pidgin Fortran language. However, for fully 90% of our computational work, the low IQ of the BRF has been more than counter-balanced by its being on-line. The magnets we build have a long cylindrical aperture surrounded by arrays of longitudinal superconducting wires and iron arranged to produce a transverse field of prescribed shape, uniform fields for bending high energy charged particle beams, and quadrupole fields for focusing. The field in the aperture is expressed, usually, in terms of the coefficients of the Taylor's expansion -- the "multipole coefficients". Point values of the field vector are also of interest, especially within the windings, as the magnitude of the field determines the allowable current. Many small programs have been developed to analyze both the two- and three- dimensional fields produced by various kinds of arrays of conductors. Some programs have the ability to vary a number of geometric parameters automatically in such a way as to drive the same number of multipole coefficients to zero. The on-line feature is especially handy, as such iterative calculations must often be cajoled into convergence.

INTRODUCTION

Particle accelerators employ electromagnets to steer and confine the particle beam. Recently, considerable attention has been devoted to the study and development of superconducting magnets for accelerators, and for the experimental beam lines external to the accelerators. Superconducting magnets have already been used on experimental beam lines, but they have not yet been utilized in an accelerator. We are currently designing the magnets for a small accelerator and storage ring, the Experimental Superconducting Accelerator Ring (ESCAR), which probably will be the first such machine to employ superconducting main-ring elements.

In 1969, the Lawrence Berkeley Laboratory designed an interactive computer system -- a somewhat mentally retarded system of quite limited capability, but one that was willing and eager -- called the BRF (Berkeley Remote Facility). Since the inception of that system, I have used it almost to the exclusion of LBL's sophisticated-but-clumsy batch-processing system for solving the various magnet engineering problems I have encountered.

While there are many kinds of engineering problems associated with superconducting magnets, I will confine the discussion to the prediction of the magnetic fields produced by the kinds of superconducting magnets used in accelerators, and the inverse problem of designing a magnet to produce a particular magnetic field shape.

THE BRF SYSTEM

The BRF system is a mini-computer subset of the Lawrence Berkeley Laboratory CDC 6600/7600 complex. A Teletype terminal serves as the input/output device, and operation is interactive. The programming language is a pidgin Fortran. All arithmetic is done in floating point. Singly or doubly subscripted arrays can be specified; the maximum number of words that can be stored in arrays is 1000. Only 10 subscripted variables may be used. The maximum number of variable names, including simple variables, subscripted variables, and numerical constants, is 60. Input and output formats are fixed. Jumps can be accomplished only via DO, GO TO XX, or IF(...) GO TO

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XX statements. Statement functions and subroutines are not permitted, with the exception of two library subroutines: one for matrix multiplication, the other for matrix inversion. Both are limited to matrices of size 10×10 . A limited number of library functions are available.

While its limitations are severe, those limitations are far outweighed, for many purposes, by its handiness. Storage, retrieval, and modification of programs are rapidly affected. Many of the BRF system's liabilities appear as assets from a different viewpoint: one is denied the freedom to specify input and output formats, but on the other hand, one is not required to specify them. While the BRF system is a unique one, it is somewhat representative of many of the mini-computers that stand on the middle ground between the pocket calculator and the super-computer

With such severe limitations, one can scarcely afford the luxury of sloppy programming. One cannot store vast arrays of numbers, then print out the whole mess at the end. Instead, one is often forced to print results as they are generated so that the storage arrays can be used again. Since the printing rate is not exactly "fast", one seldom prints out garbage he doesn't need. On the other hand, one must sometimes re-calculate a quantity simply because there is no name left by which to address it, and no pigeon hole left in which to store it.

But, when one must resort to tricky and time-consuming programming to circumvent the inherent deficiencies of the system, it is long past time to revert to batch processing, or application of a more sophisticated (and perhaps clumsy) interactive system. Even under those conditions, it is often profitable to de-bug subsets of a large program on a system such as the BRF.

MAGNETIC FIELDS

KINDS OF MAGNETS

The particular kinds of magnets under consideration are generally cylindrical and have a large ratio of length to transverse dimension. The magnetic field is transverse, not axial as in a solenoid. The winding is placed close to the aperture where it will have the greatest effect. Since superconducting magnets have high field strengths, iron situated near the aperture would saturate and do little good,

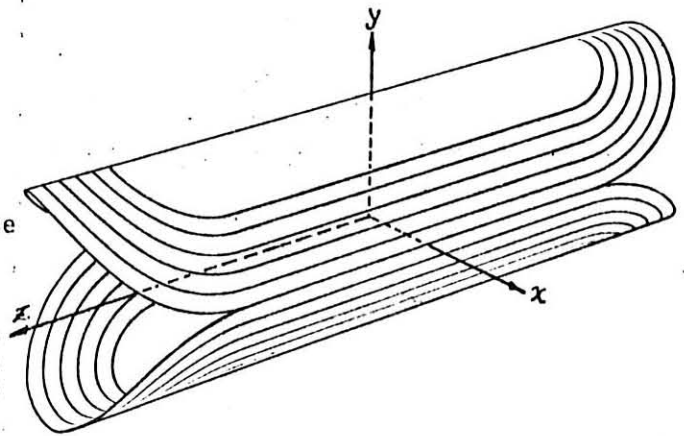


Fig. 1. Schematic illustration of coil for a dipole magnet.

so the iron flux return path is placed outside the winding. The quality of the magnetic field is dominated by the positioning of the coils and is only secondarily affected by the placement and shaping of the iron. Figure 1 shows, schematically, a winding for such a magnet and defines the coordinate system. Such a winding produces a vertical magnetic field: a "dipole" field, in the jargon of the trade. Figure 2 shows the coil structure for a magnet built in our development laboratory. Figure 3 shows the pattern of flux lines characteristic of a quadrupole magnet

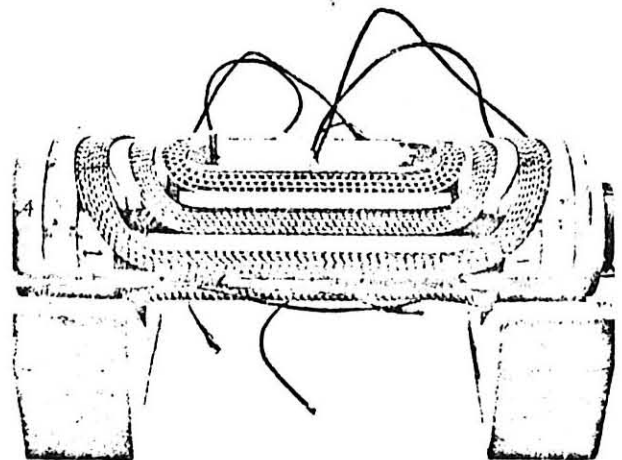


Fig. 2. Coil for small superconducting dipole magnet.

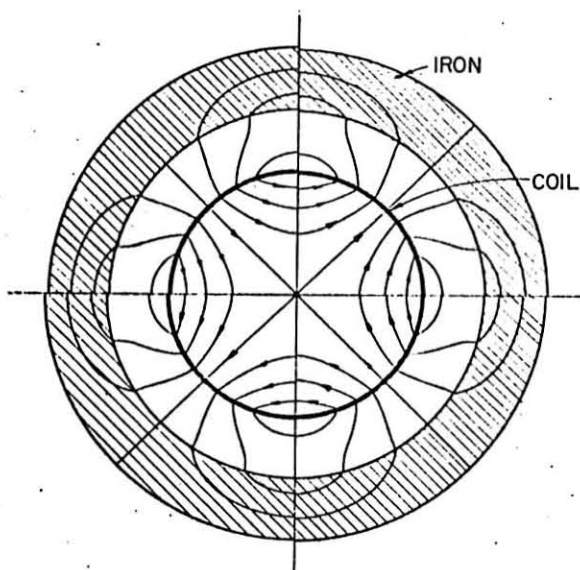


Fig. 3. Cross section of quadrupole magnet having thin winding with $\cos 2\theta$ current distribution.

MAGNETIC FIELD REPRESENTATION

The magnet user is concerned with the characteristics of the magnetic field in the magnet aperture. The magnetic field can be represented in several ways. One representation is a two- or three-dimensional map of the magnetic field vector. A more useful representation is a map of the deviation of the local field vector from some specified ideal field distribution. Yet another representation -- a rather fashionable one -- is to express either the two-dimensional field, or an integral form of the three-dimensional field, by the coefficients of a series. For a two-dimensional field, this series takes the form:

$$\begin{aligned} B_x(r, \theta) &= \sum_{n=1}^{\infty} C_n (r/\rho)^{n-1} \sin[(n-1)\theta + \alpha_n] \\ B_y(r, \theta) &= \sum_{n=1}^{\infty} C_n (r/\rho)^{n-1} \cos[(n-1)\theta + \alpha_n] \end{aligned} \quad (1)$$

where

r, θ = coordinates of point at which field is evaluated.

B_x, B_y = cartesian components of the field vector.

ρ = arbitrary normalizing radius.

α_n = a phase angle.

C_n = "multipole coefficient"; the magnitude of the field vector at radius $r = \rho$.

FIELD CALCULATION

Usually the magnet user wants a magnet that produces a pure, say, "quadrupole" field ($n = 2$). The allowable aberrations are usually expressed in terms of the allowable values of the multipole coefficients other than the desired one, or some combination of them (such as the sum of the absolute values). To determine the multipole coefficients of the field, one sometimes calculates a map of the B vector (or its scalar or vector potential) and then, using some fitting technique, determines the multipole coefficients. More often, multipole coefficients can be calculated directly. For example, for a single filament perpendicular to the x,y plane, carrying a current I, and surrounded by a cylinder of infinitely permeable iron, the multipole coefficients are:

$$C_n = \frac{\mu_0 I}{2\pi} \rho^{n-1} [1 - (a/b)^{2n}] a^n \cos n\phi \quad (2)$$

where: μ_0 = permeability of free space.

a, ϕ = conductor coordinates.

b = radius to the inside of the iron.

This equation can be integrated analytically for various simple configurations: for example, a thick or thin cylindrical shell of finite angular extent, having a uniform current density, or one which varies sinusoidally. More often the integration is performed numerically.

The fields in the end regions of the magnets are certainly not two dimensional. However, consider the following integrals of the field:

$$\int_{-\infty}^{\infty} B_x dz, \quad \int_{-\infty}^{\infty} B_y dz$$

where the integration is performed along lines parallel to the z-axis. It is mathematically legitimate to express such fields in terms of equations having the form of Eq. (1) but with the field components replaced by the corresponding integrals; the field integrals are two-dimensional.

Furthermore, Mother Nature has provided us with a convenient law: the field integrals bear the same relationship to similarly defined current integrals as the fields bear to the currents in the two-dimensional case. The contribution of a small current element to the multipole co-

efficients representing the field integrals is obtained simply by replacing I in Eq. (2) by $I dz$, where dz is the length of the projection of the current element on the z -axis. Often the integrals of the three-dimensional field are of greater interest to the magnet user than the details of the field. It is a great convenience to be able to calculate those integrals directly using simple two-dimensional methods.

MAGNET DESIGN

THE PROCEDURE

One can usually adjust some of the parameters of the coil configuration to minimize the deviations of the magnetic field from some desired field distribution. The desired field distribution is usually one corresponding to a particular multipole coefficient -- a pure quadrupole field, for example. The magnitude of all other multipole coefficients is, ideally, zero. One design procedure is to adjust the coil parameters to make a certain number of multipole coefficients exactly zero. The number that can be reduced to zero is equal to the number of parameters that can be varied.

Let x_1, x_2, x_3 represent the initial values of three adjustable parameters, and C_1, C_2, C_3 represent the initial values of three multipole coefficients that are to be reduced to zero. (Here, the subscripts of C are simply serial numbers, not harmonic order indexes.) The changes in the multipole coefficients caused by changes in the values of x may be approximated by three simultaneous equations of the form

$$\Delta C_j = -C_j - \frac{\partial C_j}{\partial x_1} \Delta x_1 + \frac{\partial C_j}{\partial x_2} \Delta x_2 + \frac{\partial C_j}{\partial x_3} \Delta x_3, \quad (3)$$

We solve the set of simultaneous equations -- or in classier language, we invert the matrix -- to obtain the values of Δx . Then as a second approximation we try values $x'_j = x_j + \Delta x_j$. (Sir Isaac Newton knew about this.) Fortunately, the BRFS system's crowning glory is a matrix inversion subroutine.

For two-dimensional fields, we adjust the coil positions, in the x, y plane, or the currents. We can also adjust the lengths of the coil elements to reduce certain multipole coefficients of the field integrals to zero. In the latter case, the equations are linear, so the solution is

obtained upon the first iteration. Occasionally, however, the mathematical "solution" requires coil sections that overlap.

AN APPLICATION

Figure 4 shows the cross section of a magnet having coils in the form of rectangular blocks of conductors. A quadrupole magnet is illustrated, but the program is applicable to multipole magnets of any order.

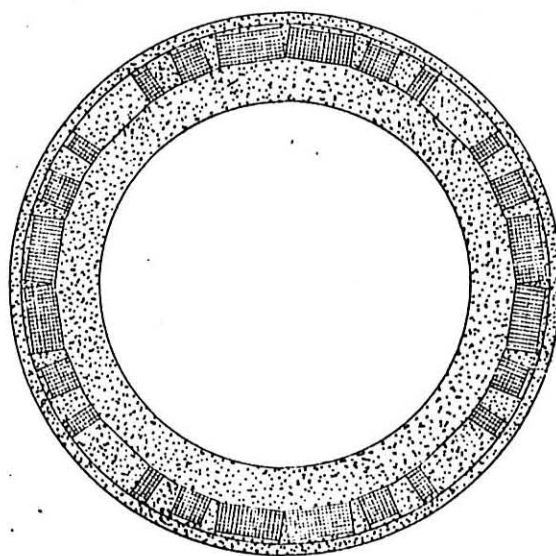


Fig. 4. Cross section of ~~one pole of~~ a quadrupole magnet, a preliminary design for ESCAR.

The initial configuration is an approximation to a known ideal one. We will hold the positions of the larger current blocks fixed and change the angular positions of the other blocks, in symmetrical fashion, according to the iterative procedure outlined earlier. The program will work for at least 10 current blocks per half pole.

The conductors of real magnets are not infinitesimal filaments, of course, but for the purpose at hand the finite conductor may be represented adequately by a single filament or, at most, a few filaments.

Such magnets often have the type of symmetry illustrated in Fig. 5. For a set of filaments arranged with the kind of symmetry shown, Eq. (2) yields:

$$C_n = \frac{2m\mu_0 I}{\pi} \rho^{n-1} [1 + (a/b)^{2n}] a^n \cos n\phi \quad (4)$$

for $n = m(1, 3, 5, \dots)$, and

$$C_n = 0 \text{ for } n \neq m(1, 3, 5, \dots)$$

where m is the number of pole pairs. So, by calculation of the multipole coefficients for the conductors associated with one-half of one pole, we obtain the field characteristics for the entire magnet.

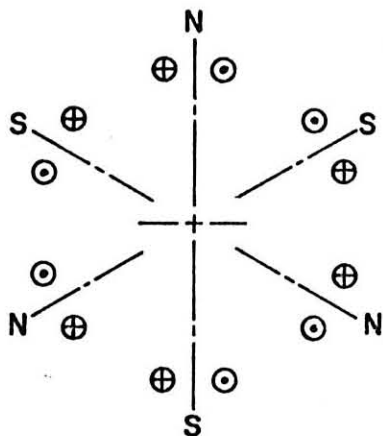


Fig. 5. Sextupole array of current filaments, one filament per half pole, having "folding" symmetry.

At this point we would be tempted simply to calculate the contribution of each filament in each current block to each multipole coefficient, then add them. Each iteration of the calculation, after each block is moved a small amount, would require a complete recalculation.

However, there are computational and conceptual advantages to representing the angle ϕ as the sum of two angles, α and θ , where θ is the position of a reference line for each block, and α is the angle of the filament from the reference line. For a block of filaments having a radial centerline, as in the present case, there are further advantages to letting the centerline be the reference line. The final form is:

$$C_n = \frac{4m\mu_0 I}{\pi} \rho^{n-1} \left\{ \sum_1 [1 + (a/b)^{2n}] a^n \cos n\alpha + \frac{1}{2} \sum_2 [1 + (a/b)^{2n}] a^n \cos n\theta \right\} \quad (5)$$

where \sum_1 is summed over one member of each symmetrical pair, and \sum_2 is summed over each filament lying on the block centerline. Now, when a block is moved, all that changes is $\cos n\theta$; the time-consuming summation remains unchanged.

The BRF program that performs the calculation is described in the Appendix. An example of its application -- a quadrupole magnet for the ESCAR ring -- is presented. In this particular application the total memory used for all simple variables, all subscripted variables, and all constants built into the program is about 300 words, and most of those are associated with the calculation of up to 15 multipole coefficients for the final design after the block positions have been optimized.

APPENDIX

The program illustrated applies to the kind of magnet shown in cross section in Fig. 4. The program is applicable to magnets having any number of pole pairs and any number of coil rectangles.

The program varies the angular positions of all of the conductor rectangles associated with a half pole except one, in order to reduce certain multipole coefficients of the field to zero. Then the program calculates the angular positions of the inner corners of the coil rectangles to indicate whether the "solution" requires rectangles that overlap.

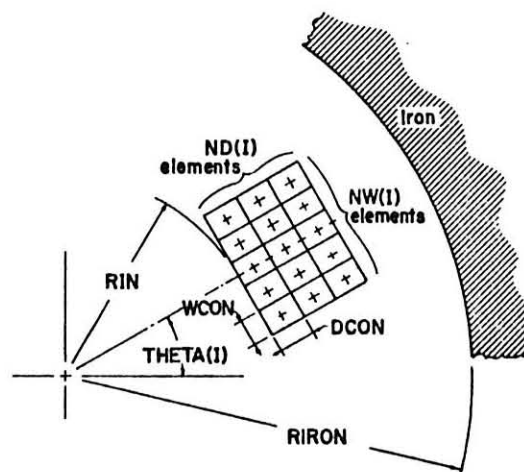


Fig. 6. Nomenclature for i -th current block as used in the BRF program.

NOMENCLATURE (See Fig. 6)

Input Data

NHARM	Number of multipole coefficients to be determined
THETA(I)	Initial angle of block centerline
DCON	Depth of Conductor
WCON	Width of conductor
ND(I)	Depth of block in units of DCON
NW(I)	Width of block in units of WCON
RIN	Inside radius of coils
CUR	Current in each conductor
SCALE	Scaling factor, see program list
RNORM	Arbitrary normalizing radius, ρ
NPAIR	Number of pairs of poles, m

Output Data

THETA(I)	Final angle of block centerline
NITER	Number of iterations
CC(1)	Multipole coefficient of order NPAIR, C_m
CC(I)	Normalized multipole coefficient, C_n/C_m , $n = 2I-1$
ALF	Angular coordinate of inner corner of block

$$C_n = \frac{4m\mu_0 I}{\pi} \rho^{-n} \left\{ \sum_{a=1}^N [1+(a/b)^{2n}] a^n \cos na + \frac{1}{2} \sum_{a=1}^N [1+(a/b)^{2n}] a^n \right\} \cos n\theta$$

Diagram labels: CNST, FEFAC, SUM, CCC, C(I,L)

PROGRAM LIST

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* BRF PROGRAM RDM191, MEUSER.
* MULTIPOLE MAGNETS. FLAT-BOTTOM PARALLEL-SIDED CURRENT
* BLOCKS. FINITE CONDUCTOR DIMENSIONS. ITERATES TO FIND
* BEST BLOCK CENTER ANGLES.
* UNITS
* PRESET UNITS ARE DEGREE, AMP., TESLA.
* FOR METERS, ENTER SCALE=1.0
* FOR CENTIMETERS, ENTER SCALE=0.01.
* FOR INCHES, ENTER SCALE=0.0254
* DIMENSIONS OF AA AND BB MUST BE ONE LESS THAN NUMBER
* OF CURRENT BLOCKS.
*****THREE BLOCKS*****
NBLK=3
DIMENSION AA(0,0),BH(0)
DIMENSION NV(3),ND(3),SUM(3,10),THETA(3)
DIMENSION C(3,10),DC(3,10),CC(10)
HEAD,DCON,WCON,CUR,SCALE
HEAD,RIN,RNORM,RINON,NPAIR
PRINT,DCON,WCON,CUR,SCALE
PRINT,RIN,RNORM,RINON,NPAIR
CNST=(1.6E-06)*NPAIR*CUR/SCALE
CONTINUE
READ,NHARM
READ,ND
READ,NV
READ,THETA
PRINT,ND,NV,THETA
DO 9 I=1,NHARM
DO 9 J=1,NBLK
SUM(I,J)=0.0
CONTINUE
DO 10 I=1,NBLK
X=RIN-0.5*DCON
LIM=ND(I)
DO 10 J=1,LIM
X=X*DCON
LIM1=NV(I)/2
Y=(NV(I)+1)*WCON*0.5
DO 12 K=1,LIM1
Y=Y*WCON
RR=SQRT(X*X+Y*Y)
ALF=ASIN(Y/RR)
DO 12 L=1,NHARM
N=(2*L-1)*NPAIR
FEFAC=1.0+(X/RINON)**(2*N)
SUM(I,L)=SUM(I,L)+FEFAC*COS(N*ALF)/RR**N
CONTINUE
IF(ABS(Y-WCON).GT.1.0E-06) GO TO 10
DO 14 L=1,NHARM
N=(2*L-1)*NPAIR
FEFAC=1.0+(X/RINON)**(2*N)
SUM(I,L)=SUM(I,L)+0.5*FEFAC/X**N
CONTINUE
DO 21 NITER=1,80
DO 20 I=1,NBLK
ARG=THETA(I)*PI/180.0
DO 20 L=1,NHARM
N=(2*L-1)*NPAIR
CCC=CNST*RNORM**(N-1)*SUM(I,L)
C(I,L)=CCC*COS(N*ARG)
DC(I,L)=-N*CCC*SIN(N*ARG)
CONTINUE
DO 28 L=1,NHARM
CC(L)=0.0
CONTINUE
DO 30 I=1,NBLK
DO 30 L=1,NHARM
CC(L)=CC(L)+C(I,L) — TOTAL CL FOR ALL BLOCKS.
CONTINUE
IF(ABS(CC(2)).LT.(1.0E-10))GO TO 42
DO 25 L=2,NBLK
BB(L-1)=CC(L)
DO 25 I=2,NBLK
AA(L-1,I-1)=DC(I,L)
CONTINUE
CALL SLV(AA,X,BB) MATRIX INVERSION, SOLUTION IS BB.
DO 26 L=2,NBLK
THETA(L)=THETA(L)+BB(L-1)*180.0/PI
IF(ABS(THETA(L)).GT.(90.0/NPAIR))GO TO 42
CONTINUE
CONTINUE
PRINT,NITER — ITERATIONS
PRINT,THETA — FINAL ANGLES
PRINT,CC(1) — FUNDAMENTAL COEF.
DO 55 L=1,NHARM
CC(L)=CC(L)/CC(1)
CONTINUE
PRINT,CC — NORMALIZED COEFFICIENTS.
DO 40 I=1,NBLK
X=NV(I)*WCON*0.5
Y=ATAN(X,RIN)*180.0/PI
ALF=THETA(I)-Y
PRINT,ALF
ALF=THETA(I)+Y
PRINT,ALF
CONTINUE
GO TO 1 — TAKE ANOTHER WHACK AT IT... TRY
DIFFERENT THETA(I), OR DIFFERENT
VALUES FOR ND OR NW.

```

CHARGE FOR A DIFFERENT NUMBER OF BLOCKS.

INPUT DATA IS ENTERED AND PLAYED BACK.

JUMPS TO HERE UPON COMPLETION.

MORE INPUT DATA, AND INSTANT REPLAY. THIS IS THE DATA MOST LIKELY TO BE CHANGED UPON SUBSEQUENT EXECUTIONS.

DETERMINES THE PROPERTIES OF EACH BLOCK WITH RESPECT TO ITS OWN CENTERLINE: THE MAIN SUMMATION.

JUMPS TO TERMINATOR IF NW IS EVEN; OTHERWISE ADDS PROPERTIES OF ELEMENTS ON BLOCK CENTERLINE.

CL OF BLOCK I, DC_L/∂θ_L.

ITERATION LOOP... TWENTY IS PLENTY.

JUMPS OUT UPON CONVERGENCE, A SIMPLE BUT ADEQUATE CRITERION.

SOLUTION IS BB.

CRASHES IF IT DIVERGES.

ANGLES OF INSIDE CORNERS... DO BLOCKS OVERLAP???

OUTPUT

```

XEQ!
BEGIN XEQ
ENTER... DCON, WCON, CUR, SCALE,
.062,.034,500,.0254!
ENTER... RIN, RNORM, RIRON, NPAIR,
3.8,2.66,6.3,2! CONDUCTOR SIZE.
DCON= 0.062 WCON= 0.034 CUR= 500.0 SCALE= 0.0254 FOR INCHES.
RIN= 3.8 RNORM= 2.66 RIRON= 6.3 NPAIR= 2.0
ENTER... NHARM, 101 INSIDE RAD. P b FOR QUADRUPOLE
ENTER... ND, MAGNET.
8,8,8!
ENTER... NJ,
32,16,8!
ENTER... THETA,
8.32,22,33!
ND 8.000000 8.000000 8.000000 } LAYERS AND TURNS PER
NW 32.000000 16.000000 8.000000 } LAYER IN EACH BLOCK.
THETA 8.320000 22.000000 33.000000 - STARTING ANGLES.
NITER= 5.0 - ITERATIONS.
THETA 8.320000 22.66234 34.14813 - FINAL ANGLES.
CC(1)= 2.13561376 - FUNDAMENTAL M'POLE COEF.
CC 1.000000 -4.44E-16 -1.11E-16 -1.74E-04 -4.20E-05 }
CC -3.49E-05 -2.41E-05 6.06E-06 2.90E-07 -2.59E-07 } NORMALIZED
ALF= 0.17301118 } 1ST BLOCK M'POLE COEFS.
ALF= 16.4669888 }
ALF= 18.5681528 } 2ND
ALF= 26.7565305 }
ALF= 32.0984174 } 3RD
ALF= 36.1978393 }
ENTER... NHARM, 2 IT'S FUN, DO IT AGAIN!
    
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